MODULE THREE

This module addresses graphing linear equations.

SC Academic Elementary Algebra Standards:

EA-4.6 Represent linear equations in multiple forms (including point-slope, slope-intercept, and standard)
EA-5.1 Carry out a procedure to graph a line when given the equation of the line.
EA-5.2 Analyze the effects of changes in the slope, *m,* and the *y*-intercept, *b,* on the graph of $y = mx + b$.
EA-5.3 Carry out a procedure to graph the line with a given slope and a *y*-intercept.
EA-5.4 Carry out a procedure to graph the line with a given slope passing through a given point.
EA-5.5 Carry out a procedure to determine the *x*-intercept and *y*-intercept of lines from data given tabularly, graphically, symbolically, and verbally.
EA-5.6 Carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally.
EA-5.7 Apply the concept of slope as a rate of change to solve problems.

Lesson # 1
Topic: Graphing linear equations using a table of values
Standard (s): EA – 5.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assesses on this prior knowledge.

- **Continuum of Knowledge**
  - In 6th grade, students locate ordered pairs on a coordinate grid (6-4.1). In 7th grade, students understand slope as a constant rate of change (7-3.2) and use inverse operations to solve equations (7-3.3). In Grade 8, students translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1) and classify relationships between two variables in graphs, tables, and/or equations as either linear or nonlinear (8-3.5).
  - In Elementary Algebra, students graph a line when given the equation of the line**.**
  - Students use this skill when graphically solving simultaneous equations or systems of linear inequalities in Intermediate Algebra (IA-2.2).

- **Taxonomy Level**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  coordinate plane
  x-axis
  y-axis
  quadrant
  coordinate(s)
  intercept(s)
  table of values
  evaluate

II. Teaching the Lesson

In this lesson, students are graphing linear equations using a table of values. Although students have done this procedure in 8th grade, a brief review of what are the characteristics of linear data using the multiple representations

may be needed. For example, asking questions like how to do I know this equation represents linear data? How do I know that this graph represents linear data? How do I know that this table of values represents linear data? Examining the data in the table to verify linearity is an excellent lead in to the concept of constant rate of change. Students will also be translating linear equations into multiple forms; therefore, the next lesson on EA – 4.6 may need to be integrated into this lesson or taught as a separate lesson.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Understand how to graph a line from an equation given in any form.
  - Create a table of values
  - Translate linear equations to multiple forms

Examples of Essential Tasks
These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

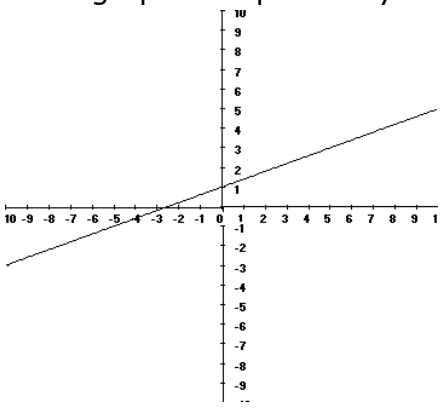
Graph the equation $y = 4 - \frac{2}{3}x$ using a table of values.

Graph the equation $5x + y = 7$ using a table of values.

Graph the equation $2x + 5y - 11 = 0$ using a table of values.

Select a graph to represent $y = 0.4x + 1$

A.



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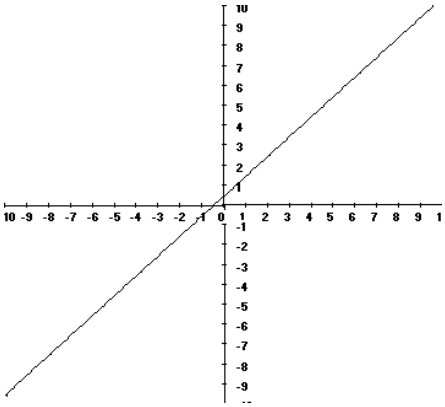
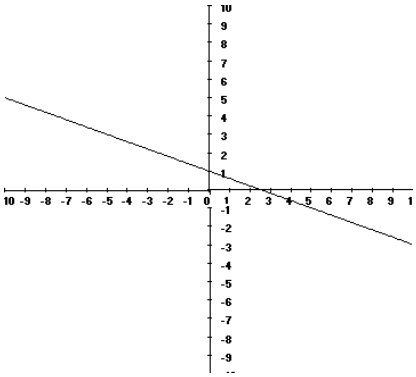
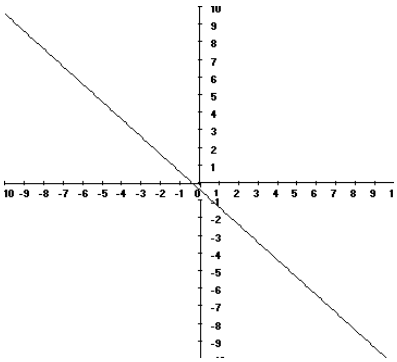


C. 

D. 

Non-Essential Learning and Understanding
It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

Graph lines whose equations include irrational numbers.

Examples of Non-Essential Tasks
The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

Sketch the graph of $3x + 2y + 4z = 12$

Sketch the graph of $2 = \sqrt{5}\,x + \sqrt{2}\,y$

Misconceptions/Common Errors
- Students may have difficulty evaluating expressions.
- Students may have having difficultly solving equations for y.
- Students may assume that the scale of the graph is always one unit.

Technology Note
- Students may use technology for complex computation.
- Students may use technology for graphing but must be able to transfer the graph to paper.
- Students may use technology to verify graphs produced by hand. Students should be aware that the scale of a graphing utility may make the line appear to be more or less steep than the graph produced by hand.

Assessment Guidelines
The objective of this indicator is to carry out a procedure to graph a line given its equation. Therefore, the primary focus of the assessment should be for students to carry out such procedures. For this indicator, assessment items should be written that encourage students to generate a table of values for x and y.

Lesson # 2
Topic: Representing linear equations in various forms
Standard (s): EA – 4.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 8th grade, students identify the coordinates of the *x*- and *y*-intercepts (8-3.6). Students also translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1). The forms of these linear equations include the slope-intercept form and the standard form which will vary from text to text.
  - In Elementary Algebra, students represent linear equations in multiple forms (including point-slope, slope-intercept, and standard).
  - This essential skill is necessary in all subsequent study of mathematics.

- **Taxonomy**
  2.1-C
  Cognitive Process Dimension: Understand
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Point-slope form
  Slope-intercept form
  *Standard form (will vary from text to text)
  Equivalency

II. Teaching the Lesson

*In this lesson, students translate linear equations into multiple forms. At this point, students are aware of slope-intercept form and may or may not have been introduced to standard form. * As a result of standard form varying from text to text, the focus should be on students using their understanding of equivalency and algebraic manipulation. In addition to becoming fluent in translating linear equations into multiple forms, students gain a conceptual understanding of equivalency.*

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

- Recognize and use forms of linear equations such as point-slope, slope-intercept and standard.
- Use algebraic techniques to translate linear equations from one form to another

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
  - Represent the equation $2x + 5y = -10$ in slope-intercept form.
  - Represent the equation $-3x - 6y - 9 = 0$ in slope-intercept form.
  - Which of the following is an equivalent form of $y = \frac{-2}{3}x - 4$?

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - None noted

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - None noted

- **Misconceptions/Common Errors**
  Students may not understand that linear equations represented in different forms can represent the same line. For example, $y = -2x + 5$ is the same line as $2x + y = 5$.

- **Technology**
  Many graphing utilities require that the equation be converted to slope-intercept form before entering the equation into the calculator. Exploring the graphs of linear equations in various forms may be used to address the misconception that linear equations represented in different forms can represent the same line.

III. Assessing the Lesson

***Assessment Guidelines**: This objective of this indicator is for the student to represent linear equations in multiple forms. Therefore, the primary focus of the assessment should be for students to become fluent in the algebraic techniques necessary to translate equations to multiple forms.*

- **Assessment Item Examples**

  - What is the slope-intercept form of 2x – y = 10?
    A. y = x – 2
    B. y = –2x +10
    C. y = 2x + 12
    D. y = 2x + 10

  - What is the equivalent form for $y = \frac{2}{5}x - 1$?

    A. $5x + 2y = -5$
    B. $2x + 5y = -5$
    C. $5x - 2y = 5$
    D. $2x - 5y = 5$

  - Find an equivalent form of y = -3/7 x + 5.
    A. 7y + 5 = -3x
    B. 7y + 35 = -3x
    C. 7y -5 = -3x
    D. 7y – 35 = -3x

  - Find an equivalent form of y – 4 = -6(x – 2)
    A. 6x +y + 8 = 0
    B. 6x + y = 8
    C. 6x + y = 6
    D. 6x + y + 6 = 0

IV. Resources

Lesson # 3
Topic: Determining the slope using the multiple representations
Standard (s): EA – 5.6

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7th grade students understand slope as a constant rate of change (7-3.3). In Grade 8, students identify the slope of a linear equation from a graph, equation, and/or table (8-3.7).
  - In Elementary Algebra, students carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally.
  - In Pre-calculus, students carry out a procedure to compute the slope of a line when given the angle of inclination of the line (PC-5.15).

- **Taxonomy**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  Evaluation
  Simplification
  Substitution

II. Teaching the Lesson

In this lesson, students determine the slope. Examining slope through the multiple representations give students a deeper conceptual of the concept. Students will use this skill in Module 4 to write linear equations.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Given at table containing solutions to a linear equation, use two of those values to calculate the rise and run of the line and divide the rise by the run to calculate the slope.

- Given a graph of a line, identify two points on the line, identify the x and y coordinates of those points, use those values to calculate the rise and run of the line and divide the rise by the run to calculate the slope.
- Given any form of an equation of a line, rewrite the equation in slope intercept form and identify the slope of the line.

- Given a verbal description of a constant rate of change between two variables, recognize this as the slope of a linear relationship.

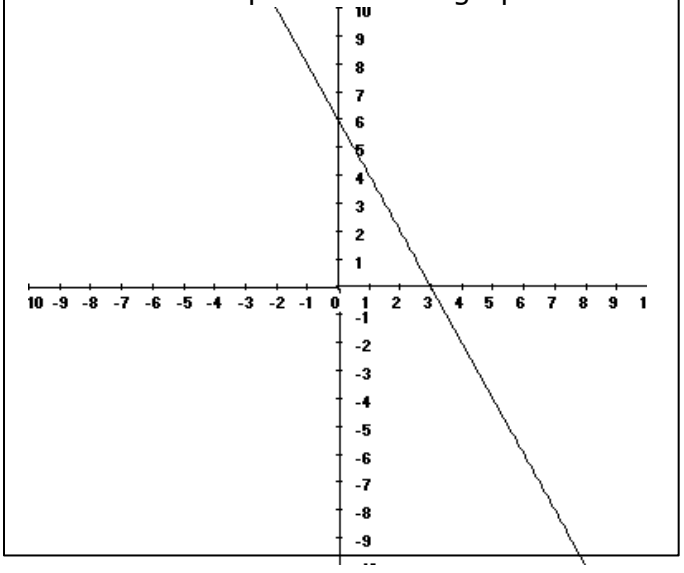
- **Examples of Essential Tasks**

  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - The table below contains some values of a linear equation. What is the slope of the line?

X	-2	-1	0
Y	-8	-5	-2

  - What is the slope of the line graphed below?



  - What is the slope of the line with the equation 3x + 2y = 7?
  - What is the slope of the function f(x) = -x + 4?
  - The slope of a roof describes the steepness of a roof. If a roof rises 9 feet for every 15 feet of run, what is the slope of the roof?

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - Find the slope of a line whose coordinates involve irrational numbers.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - The slope of the line with an equation $\sqrt{5}x+4y = 7$.

- **Misconceptions/Common Errors**
  - Students may confuse the slope with the y-intercept.
  - Students may confuse the x-axis with the y-axis (or the x-intercept with the y-intercept.
  - Students may invert the slope, using the change in x as the numerator and the change in y as the denominator of the slope.
  - Students may assume that the scale of the graph is always one unit.

- **Technology**
  Students may use technology to check computations.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to carry out a procedure to determine the slope of a line from data given tabularly, graphically, symbolically, and verbally. Therefore, the primary focus of the assessment should be for students to carry out such procedures.

- **Assessment Item Examples**

  - The table below shows some values of a linear equation. What is the slope of the line?

X	Y
-3	10
0	5
3	0
6	-5

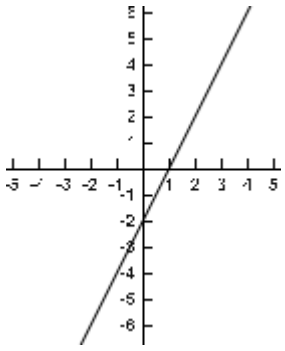
A. $-\frac{3}{5}$

B. $-\frac{5}{3}$

C. $\frac{5}{3}$

D. $\frac{3}{5}$

  - What is the slope of the line graphed below?



A. 1

B. -1

C. 2

D. -2

  - What is the slope of the line 10x + 5y = 20?

A. -2

B. 2

C. 10

D. -10

- A ramp is built for racing cars at the derby. For every 21 feet of run, the ramp rises 7 feet. What is the slope of the ramp?

  A. 3

  B. -3

  C. $\frac{1}{3}$

  D. $-\frac{1}{3}$

IV. Resources

Activity:

- Many students have difficulty drawing the rise over run triangle so using the stairs in your schools to illustrate rise over run may be helpful.
- When determining the slope from a table of values, leave the points in the table. Let the student select two points, determine the change in y and the change in x then write the ratio. Although students are not formally writing out the formula, they still have a conceptual understanding of the process.

Lesson # 4
Topic: Slope as a rate of change
Standard (s): EA – 5.7

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7th grade students understand slope as a constant rate of change (7-3.3). In Grade 8, students identify the slope of a linear equation from a graph, equation, and/or table (8-3.7).
  - In Elementary Algebra students apply the concept of slope as a rate of change to solve problems.
  - In Pre-calculus students carry out a procedure to compute the slope of a line when given the angle of inclination of the line (PC-5.15).

- **Taxonomy**
  3.2-B
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  slope
  rise
  run

ratio
rate of change

II. Teaching the Lesson

In the previous lesson, students found the slope using procedural knowledge such as calculating the rise over run and using the slope formula. In real world situations, slope is more than rise over run. Rate of change represents how one quantity changes with respect to another quantity; therefore, a rate of change can be negative or positive. In this lesson, students gain a deeper conceptual understanding by exploring more rigorous real life applications of slope.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Recognize the slope of a line in a given linear relationship.
  - Interpret the slope of a line in a given linear relationship as a rate of change between two variables.
  - Use the slope to find the change in one variable for a given change in the other variable in a linear relationship.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - In the equation y = 4x, if the value of x is increased by two, what is the effect on the value of y?
    a)It is increased by eight
    b)It is increased by six
    c)It is increased by 2
    d)It is sixteen times the original amount
    e)It is eight times the original amount
    (NAEP released item)
  - Yvonne has studied the cost of tickets over time for her favorite sports team. She has created a model to predict the cost of a ticket in the future. Let *C* represent the cost of a ticket in dollars and *y* represent the number of years in the future. Her model is as follows.

    $$C = 2.50y + 13$$

    Based on this model, how much will the cost of a ticket <u>increase</u> in two years?

a) $5 b) $8 c) $13 d) $18 e) $26
(NAEP released item)

- Carla has a lemonade stand. Her profit is modeled by y = 0.75x – 9.85 where y is profit and x is the number of cups of lemonade sold. If she sells five more cups of lemonade today than she sold yesterday, how much more money does she make today?

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:

  Evaluate the value of the variables. The problem should focus on the rate of change.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

  - Carla has a lemonade stand. Her profit is modeled by y = 0.75x – 9.85 where y is profit and x is glasses of lemonade sold. If she sells five glasses of lemonade, what is her profit?

- **Misconceptions/Common Errors**
  Students may not understand rates (speed, growth, gas mileage) as slopes.

- **Technology**
  Use technology where appropriate.

III. Assessing the Lesson

Assessment Guidelines: The objective of this indicator is to <u>apply</u> the concept of slope as a rate of change to solve problems. Therefore, the primary focus of the assessment should be for students to apply this concept.

- **Assessment Item Examples**

  - In the equation y = 3x, if the value of x is increased by 2, what is the effect on the value of y?
    A. It is tripled.

    B. It is increased by 6.

    C. It is increased by 2.

D. It is 6 times the original amount.

- Maria has watched the price of gasoline and has created a model to predict the cost of gasoline in future months. Let *C* represent the cost of gasoline in dollars for the future and *m* represent the number of months in the future. The model is as follows:

  C = 0.35m + 3.50

  Based on this model, how much will the cost of a ticket increase in 6 months?

  A. $0.70

  B. $0.35

  C. $1.05

  D. $2.10

- Using the same formula, how much more will gasoline cost in 2 months than it does now?

  A. $.70

  B. $0.35

  C. $1.05

  D. $2.10

IV. Resources

Activity: Distance-time examples are an excellent way to illustrate rate of change. Using a CBR and the TI program Match It, students can create linear graphs by walking at a constant rate. This activity requires only one CBR, one calculator with the built in program and one overhead projector. Visit www.myscmsu.org to locate your regional mathematics and science center to find out the process for checking out a CBR.

Lesson # 5
Topic: Graphing linear equations using slope and y-intercept
Standard (s): EA – 5.3 and EA – 5.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7th grade students understand slope as a constant rate of change (7-3.2). In 8th grade, students use intercepts to locate lines in a coordinate plane (8-4.2) and translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1).
  - In Elementary Algebra, students graph the line with a given slope and a *y*-intercept.
  - In Intermediate Algebra, students carry out a procedure to graph translations and transformations of parent functions (IA-2.7 and IA-2.8).

- **Taxonomy**
  3.1-C
  Cognitive Process Dimension: Apply

Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  slope
  y-intercept
  slope-intercept form
  scale

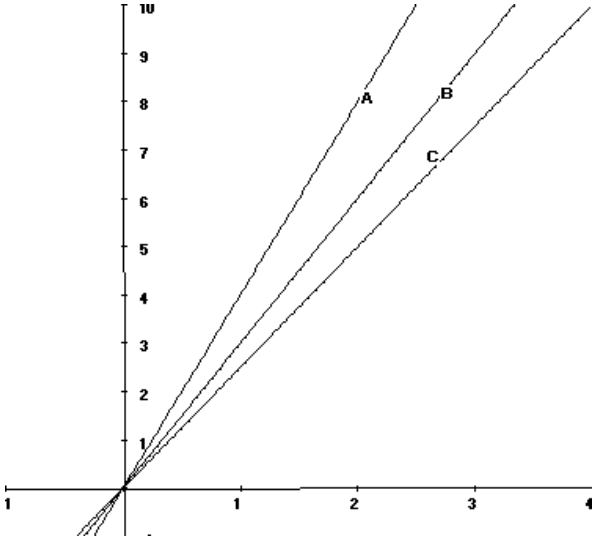
II. Teaching the Lesson

In this lesson, students graph linear equations using slope and y-intercept. Students may have difficulty knowing where to begin when graphing. Emphasizing that in slope intercept form, b stands for "begin" and m stands for "move" may clarify this process. In previous lesson, students used the staircase to model the rise over run triangle. Revisiting that process may helpful as students graph using the slope and y-intercept.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Determine the slope and y-intercept of a line from the slope-intercept form of the equation.
  - Plot on a coordinate plane the y-intercept of a line.
  - Use slope and the y-intercept to locate on a coordinate plane a second point on a line.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
  - Graph a line with a slope of -3 and a y-intercept of 7.
  - Graph the equation $y = \frac{4}{3}x + 5$
  - Graph $y - 4x = 12$
  - Graph $y = 6x + 0.75$
  - Graph $4x + 5y = 15$
  - Graph the equation $2x + 2y - 4 = x + 5$.

Tom runs at a rate of 2.5 mph. Sue runs at a rate of 3 mph and Fred runs at a rate of 4mph. Which line in the following graph represents Sue's run?



- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - ○Graph a line with a slope that is an irrational number.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - ○Graph the line with slope π/2 and y-intercept 4.

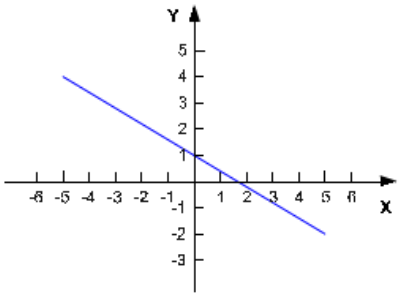
- **Misconceptions/Common Errors**
  - ○ Students may confuse the slope and the y-intercept.
  - ○ Students may assume that the scales of the x and y axes are always single units.

- **Technology**
  Students may use technology for graphing but must be able to transfer the graph to paper.
  Students may need instruction and practice in selecting appropriate windows and scales when creating graphs of linear equations and linear functions using a graphing utility.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to graph the line with a given slope and a y-intercept**.** Therefore, the primary focus of the assessment should be for students to carry out such procedures. In addition, teachers may write assessment items that ask students to <u>select</u> a graph that is appropriate for an equation or an equation that is appropriate for a graph.*

- **Assessment Item Examples**
  - Which of the following equations represents the graph below?



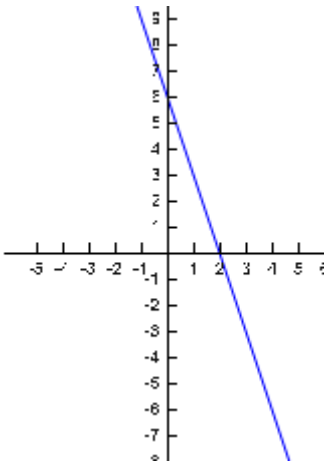
A. $y = -\frac{3}{5}x + 1$

B. $y = \frac{3}{5}x + 1$

C. $y = -\frac{5}{3}x + 1$

D. $y = \frac{5}{3}x + 1$

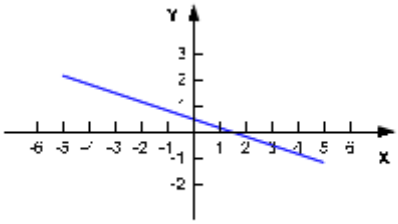
  - Which of the statements describes the graph below?



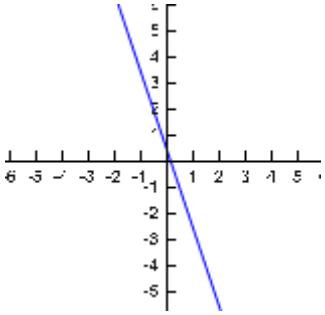
A. The slope of the line is -6 and the y-intercept is 2.
B. The slope of the line is -3 and the y-intercept is 2.
C. The slope of the line is -3 and the y-intercept is 6.
D. The slope of the line is 3 and the y-intercept is -6.

- Which of the following shows the graph of the equation 4x + 12y = 6.

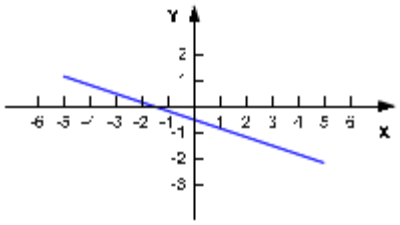
A.



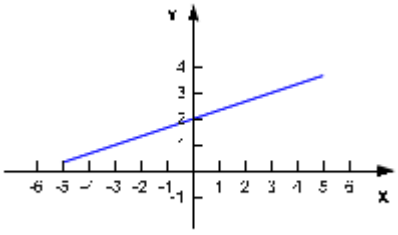
B.



C.



D.



IV. Resources

Lesson # 6
Topic: Examining the effect of changes on m and b on the graph of a linear function
Standard (s): EA – 5.2

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In 7th grade students understand slope as a constant rate of change (7-3.2). In 8th grade, students use intercepts to locate lines in a coordinate plane (8-4.2) and translate among verbal, graphic, tabular, and algebraic representations of linear functions (8-3.1).

- In Elementary Algebra, students analyze the effects of changes in the slope and the *y*-intercept on the graph of a line.
- In Intermediate Algebra, students carry out a procedure to graph transformations of parent functions other than linear functions (including $y = x,\ y = x^2,$ and $y = |x|$) (IA-2.8).

- **Taxonomy**
  4.1-B
  Cognitive Process Dimension: Analyze
  Knowledge Dimension: Conceptual Knowledge

- **Key Concepts**
  slope
  y-intercept
  slope-intercept form

II. Teaching the Lesson

In this lesson, students examine how changes in the slope and y-intercept affect the graph of a linear function. These changes can also be illustrated through real world connections that provide students a context in which to examine these changes. For example, analyzing the cell phone plans of two companies where the 1) base fee is the same and the cost per minute is different and where the 2) base fee is different and the cost per minute is the same.

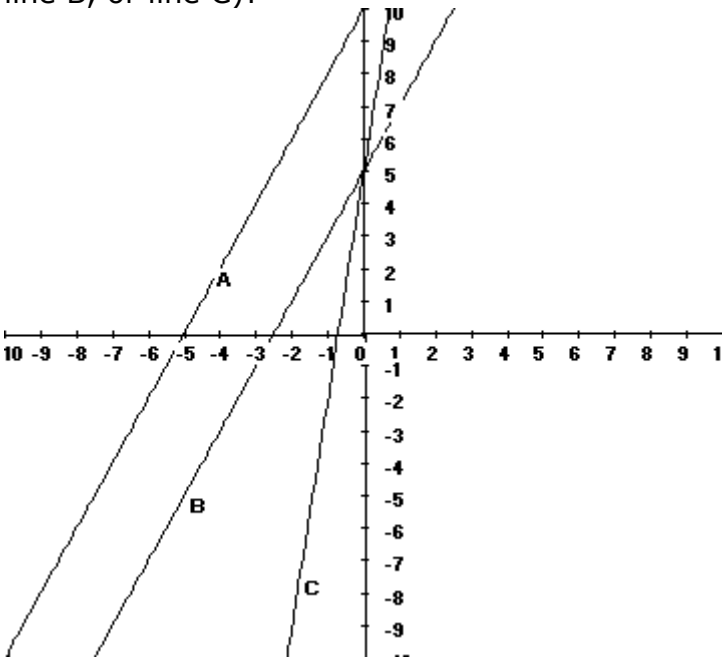
- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - Determine the slope and y-intercept of a line, given its equation in slope-intercept form.
  - Understand how the value of the slope affects the steepness of the line.
  - Understand how the value of the y-intercept affects where the line crosses the y-axis.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

  - What will happen to the graph of y = 2x + 5 if the value of the y-intercept decreases by two units?
  - What will happen to the graph of y = 2x + 5 if the value of the slope increases by two units?

- Graph y = 2x + b, where b = {-2, -1, 0, 1, 2}. Write a sentence to summarize the effect of a change in b on the graph of the linear equation.
- How do the graphs of $y = x + 3$ and $y = x + 5$ differ?
- How does changing the constant c in $y = x + c$ affect the graph?
- What does decreasing the constant c by 2 units in an equation of the form $y = x + c$ do to its graph?
- Study the graphs of $y = x + c$ for c = 1, c = 2, c = 3, c = -1 and c = -2. Write a sentence describing how changing the constant c in $y = x + c$ affects the graph.

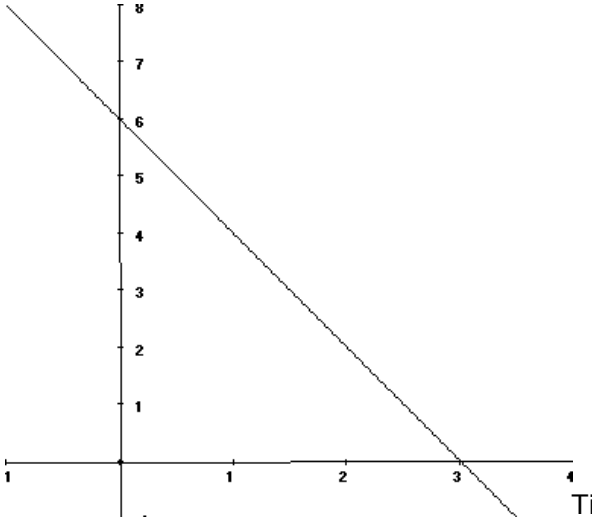
If the slope of the line whose equation is y = 2x + 5 is increased by 5 units, which of the following is the graph of the new equation (line A, line B, or line C)?



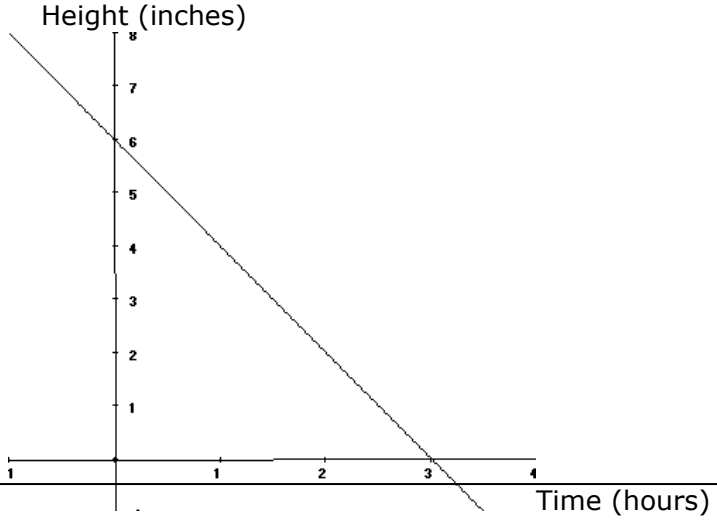
- Write a sentence to compare the graphs of y = 2x + 3 and y = 4x + 3. *(Possible answer: Both lines pass through (0. 3) but y = 2x + 3 is not as steep as y = 4x + 3)*
- Describe the change that would occur if y = 2x + 4 is changed to y = 2x – 4. *(Possible answer: The lines would be parallel but y = 2x + 4 would have a y-intercept of positive 4 and y = 2x – 4 would have a y-intercept of negative 4.)*

Below is a graph of the height of a six inch candle that burns at a rate of two inches per hour. How would the graph change if the candle burned at a rate of three inches per hour?

Height (inches)



Below is a graph of the height of a six inch candle that burns at a rate of two inches per hour. How would the graph change if the candle were 10 inches tall?



- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - ○Analyze the effects of changes in the slope, m, and the y-intercept, b, where m is an irrational number.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - ○ Sketch the graph of -1.552x – 2.233y = π
  - ○ Sketch the graph of y = $\sqrt{5}$x + 4

- **Misconceptions/Common Errors**
  Students may confuse the slope with the y-intercept.

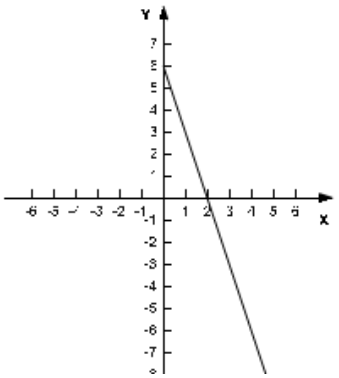
- **Technology**
  This topic may be introduced by having the students explore the graphs of parallel lines having a variety of y-intercepts or the graphs of lines with the same y-intercept and a variety of slopes to discover patterns in the changes of the graphs.
  Students may use technology when discovering the transformational patterns that occur when the slope and/or y-intercept are changed.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to <u>analyze</u> an equation to discover how the equation relates to the graph of the equation. Students should also be able to <u>analyze</u> a graph to <u>select</u> an appropriate equation. Therefore, the primary focus of the assessment should be for students to <u>analyze</u> the graph or the equation in order to connect the graph with the equation*

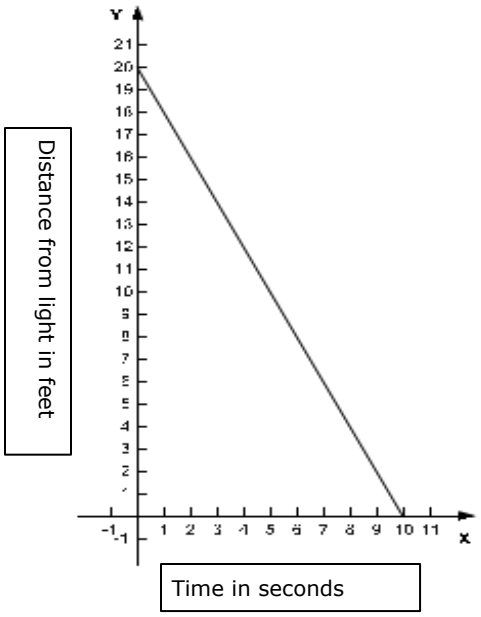
- **Assessment Item Examples**

  - The graph of y = -3x + 6 is shown below.



  - If the function is translated down 3 units, what would be the new function?
    A. y = -3x + 3
    B. y= -x + 2
    C. y= -3x – 1
    D. Y = -x + 6

- The graph below shows that Keri stands at a point 20 feet away from a light and walks towards the light at a rate of 2 feet per second.



- How would this graph change if Keri started walking at a point 50 feet from the light and maintained walking at 2 feet per second?
  A. The y-intercept will be 50.
  B. The x-intercept will be 50.
  C. The slope will be 50.
  D. The slope will be -50.

- How would the graph change if Keri started walking at 20 feet from the light and walked at a rate of 4 feet per second?
  A. The y-intercept will be 4.
  B. The x-intercept will be 20.

C. The slope would be steeper.
D. The slope would not be as steep.

IV. Resources

Lesson # 7
Topic: Graphing linear equations given the slope and a point.
Standard (s): EA – 5.4

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In grade six, students represent with ordered pairs of integers the location of points in a coordinate grid. (6-4.1) In 7th grade students understand slope as a constant rate of change (7-3.2).
  - In Elementary Algebra, students graph the line with a given slope passing through a given point**.**
  - In Intermediate Algebra, students carry out a procedure to graph translations and transformations of parent functions (IA-2.7 and IA-2.8).

- **Taxonomy**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  slope
  point-slope form

II. Teaching the Lesson

In this lesson, students build a conceptual understanding of graphing linear functions. When graphing linear equations in slope intercept form, students

use the y-intercept because it is the easiest point to use. Students need to conceptually understand that the same process can be done using the slope and any point. This emphasizes that slope is constant and that there are several points that make up a line.

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:
  - ○Understand the Cartesian coordinate plane system.
  - ○Understand that a unique line is determined by two points
  - ○After graphing a given point, use the slope to determine a second point.

- **Examples of Essential Tasks**
  These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.
  - ○ Select the graph that represents an equation with a slope of 3 and passing through the point (-4, 7).
  - ○ Graph a line that passes through (3.25, -3) and has a slope of 5.
  - ○ Graph a line that is parallel to y=3x -7 and that passes through the point (1, 2).
  - ○ Graph a line that is perpendicular to y=3x -7 and that passes through the point (1, 2).

- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - ○ Graph piecewise functions.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - ○ Graph this function: $f(x) = \begin{cases} -1 \text{ if } x < 0 \\ 1 \text{ if } 0 \geq x \geq 2 \\ 3 \text{ if } x > 2 \end{cases}$

- **Misconceptions/Common Errors**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.

- Graph this function: $f(x) = \begin{cases} -1 \text{ if } x < 0 \\ 1 \text{ if } 0 \geq x \geq 2 \\ 3 \text{ if } x > 2 \end{cases}$

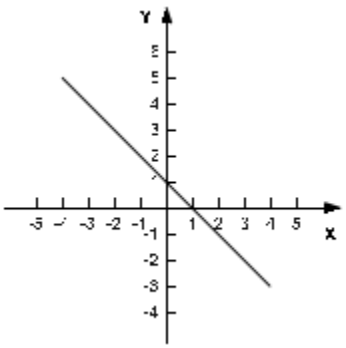
- **Technology**
  Students may use technology to verify answers after they have placed the equation in function form.

III. Assessing the Lesson

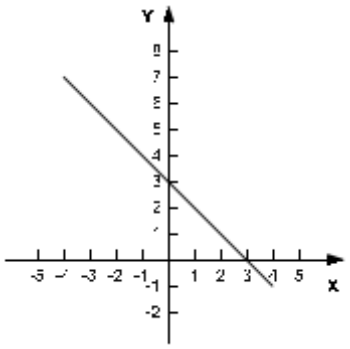
Assessment Guidelines: *The objective of this indicator is to carry out a procedure to graph the line with a given slope passing through a given point. Therefore, the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**

  - Which graph shows an equation of a line with slope -1 and passing through the point (-2, 3)?
    A.



    B.



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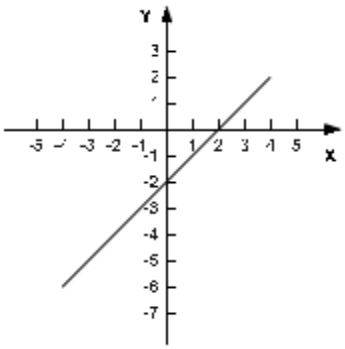
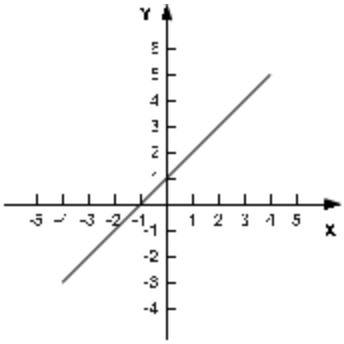
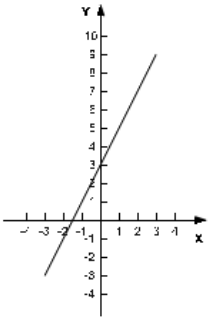


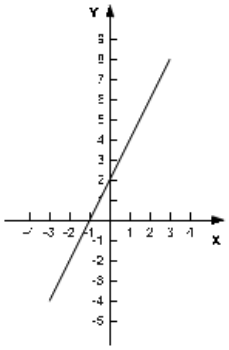
D.



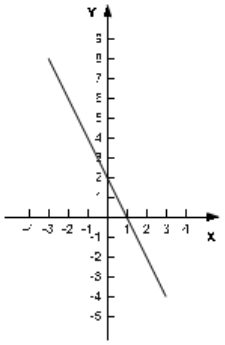
- Refer to the graphs below to answer the following two questions.
  - Which graph is parallel to y = 2x +3 and passes through the point (1, 0).
  - Which graph is perpendicular to y = 2x + 3 and passes through the point (1, 0).

A.

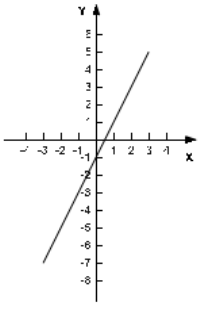




B.



C.



D.

IV. Resources

Lesson # 8
Topic: Using the x and y intercepts of linear equations to graph the equation
Standard (s): EA – 5.5 and EA- 5.1

I. Planning the Lesson

The first bullet under the Continuum of Knowledge represents student's prior knowledge and/or skills needed to meet this standard. It is recommended that students are pre-assessed on this prior knowledge.

- **Continuum of Knowledge**
  - In grade six, students represent with ordered pairs of integers the location of points in a coordinate grid. (6-4.1) In Grade 8, students translate among verbal, graphic, tabular, and algebraic representations of linear functions. (8-3.1)
  - In Elementary Algebra, students determine the *x*-intercept and *y*-intercept of lines from data given in tables, in graphs, using symbols and using words. Student understanding should exceed rote operational proficiency. Students also graph linear equations using the x-intercept and y-intercept

- In Intermediate Algebra, students carry out a procedure to determine specified points (including zeros) of polynomial functions. (IA-4.2)

- **Taxonomy**
  3.1-C
  Cognitive Process Dimension: Apply
  Knowledge Dimension: Procedural Knowledge

- **Key Concepts**
  x-intercept
  y-intercept
  x-axis
  y-axis

II. Teaching the Lesson

In previous lessons, students used slope/y-intercept and slope/point to graph a linear equation. In this lesson, students explore the x-intercept and y-intercept using the multiple representations. In addition to determining the x-intercept and y-intercept, students use their knowledge of x and y intercepts to graph a linear equation. To help with student's misunderstanding of which variable should have a value of zero in each intercept, share this football analogy. There are two teams: the X team and the Y team. When X intercepts the ball, Y has nothing so y = 0; (X,0). When Y intercepts the ball, X has nothing so X = 0; (0,Y).

- **Essential Learning and Understanding**
  It is essential for students to do the following for the attainment of this indicator:

  - Recognize the x and y-intercepts form a table of values.
  - Recognize the x and y-intercepts as the points where a line intersects each axis.
  - Substitute a value of zero for x in a linear equation to find the y-intercept.
  - Substitute a value of zero for y in a linear equation to find the x-intercept.
  - Translate from linear verbal models to algebraic models.
  - Recognize that vertical lines may have no y-intercept and that horizontal lines may have no x-intercept.

- **Examples of Essential Tasks**

These examples of essential tasks are not all inclusive. They are provided to give additional clarification of possible tasks that students should be able to successfully complete.

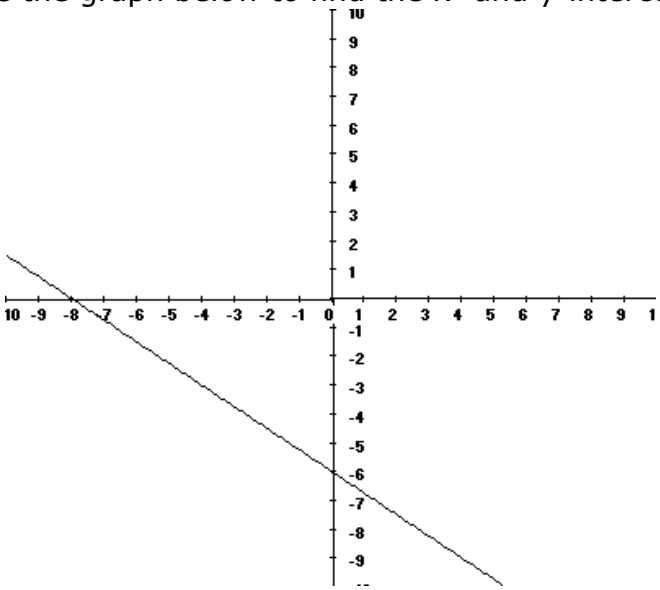
- The table of values below lists points on a line. What is the y-intercept of the line?

X	-1	0	1	2
Y	6	3	0	-3

- Select the equation that has an x-intercept of -4 and a y-intercept of 2.

a) $y = -\frac{1}{2}x - 4$
b) $y = 2x - 4$
c) $y = -\frac{1}{2}x + 2$
d) $-4x + 2y = 0$

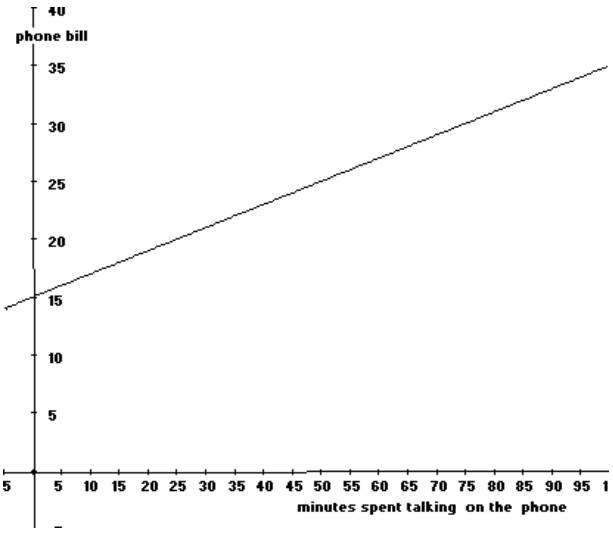
- Use the graph below to find the x- and y-intercepts.



- What is the x-intercept of the graph of $y= 2x +6$?

- What is the y-intercept of the graph of $4y - 2x = 6$?

- A cell phone provider charges a flat fee of $15 plus $.10 per minute each month. Below is the graph of a function modeling the cost of the cell phone as a function of the number of minutes used. What is the y-intercept and what is represented by the y-intercept?



- **Non-Essential Learning and Understand**
  It is not essential for students to do the following for the attainment of this indicator but could be important for the attainment of other indicators within Elementary Algebra:
  - ○Determine the intercepts of piece-wise functions.
  - ○Determine the intercepts of non-linear functions.

- **Examples of Non-Essential Tasks**
  The examples of non-essential tasks given below are not essential for the attainment of this particular indicator but could be important for the attainment of other indicators within Elementary Algebra.
  - ○ Find the y-intercept of $y = |x+2|$
  - ○ What is the x-intercept of $y = x^3$?

- **Misconceptions/Common Errors**
  - ○ Students may mistakenly substitute 0 for x to find the x-intercept (or 0 for y to find the y-intercept).
  - ○ Students may be confused when the graph of a line goes through the origin.

- **Technology**
  Many calculators have the capability to display a split screen. Displaying the graph and the table of values simultaneously may be useful when examining how the two representations are connected.

III. Assessing the Lesson

Assessment Guidelines: *The objective of this indicator is to carry out a procedure to determine the x-intercept and y-intercept of lines from data given tabularly, graphically, symbolically, and verbally. Therefore the primary focus of the assessment should be for students to carry out such procedures.*

- **Assessment Item Examples**
  - The table of values below lists the points on a line. What is the y-intercept of the line?

X	y
-2	6
0	3
2	0
4	-3

A. 3

B. 0

C. -3

D. 2

  - Which of the following equations has an x-intercept of 4 and a y-intercept of -5?

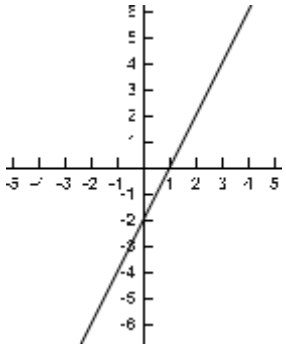
A. -5x – 4y – 20 = 0

B. 5x - 4y – 15 = 0

C. 5x – 4y – 20 = 0

D. -5x + 4y + 15 = 0

  - Use the graph below to determine the x and y intercepts.



A. x-intercept is 1, y-intercept is -2

B. x-intercept is -2, y-intercept is 1

C. x-intercept is -1, y-intercept is 2

D. x-intercept is 2, y-intercept is -1

- Find the x and y intercepts of the following equation.
  $3x - 2y = 6$
  A. x-intercept is 3, y-intercept is -2

  B. x-intercept is -2, y-intercept is -3

  C. x-intercept is 2, y-intercept is -3

  D. x-intercept is 2, y-intercept is 3

- The principal of a local high school is purchasing golf shirts for the faculty members. The cost of the shirts is modeled by the equation $C = 11.50s + 25$, where the cost (C) is a function of the number of shirts (s) purchased. What is the y-intercept and what does it mean?
  A. 25, the number of shirts purchased

  B. 25, the initial cost to set up for the shirts

  C. 11.50, the cost of one shirt

  D. 11.50, the number of shirts purchased

IV. Resources